

ABSTRACT

It is well known that estimation of the parameters, in particular frequency a complex sinusoid contaminated with noise is one of the crucial problems in the literature as the frequency estimation has been applied in many areas such as communications, instrumentation and radar. There are three significant issues which are accuracy, estimator variance and sensitivity to bias that must be taken into consideration while finding the frequency of a sinusoid. In this study, performances of various estimators in estimating the frequency of single-tone signals are evaluated. Estimators are tested against varying signal-noise-ratio (SNR) and imperfections in signal frequency. The results are presented in terms of estimator bias and estimator variances. Experimental results show that in the case of varying SNR, Quinn's estimator outperforms Macleod and quadratic interpolation estimators in accuracy whereas in the case of imperfections in signal frequency, Macleod and quadratic interpolation estimators achieve the most stable results in terms of estimator variance and bias, respectively.

KEYWORDS: parameter estimation, frequency estimation, estimator variance, estimator bias.

I. INTRODUCTION

Parameter estimation is the process of estimating one of the parameters like complex frequency components of a tone contaminated by noise. Estimating the parameters of a tone signal has numerous applications in communications, radar, instrumentation, audio, medical and others. In statistics literature, the problem is also known as harmonics estimation problem [1] whereas in signal processing framework it is known as tone parameter estimation [2].

Various methods have been proposed for this estimation task in the literature [3-9]. The application of the Discrete Fourier Transform (DFT) for the coarse estimation of noisy single frequency signals was one of the initial studies [3]. In [4], Maximum Likelihood (ML) estimator was introduced for the estimation of single-frequency complex tone from noisy observations of the signal. Quinn [5] proposed a computationally simple algorithm compared to the previous approaches by interpolating Fourier coefficients at three distinct frequencies. It was Macleod's study [6] which presented a fast algorithm for the ML estimation of the parameters of single and multiple tone signals. In literature, there are also some other approaches based on the interpolation of DFT samples [7-9]. In [8], a quadratic interpolation fit is proposed based on three DFT samples. This fit uses the magnitudes of the DFT samples. On the other hand, [9] utilizes the real part of interpolation of complex DFT samples.

In this study, we compare and present the performance results of various estimators in estimating the frequency of single-tone signals. The results are given in terms of estimator bias and estimator variance for cases of varying signal-to-noise ratio (SNR) and imperfections in the signal frequency.

II. MATERIALS AND METHODS

Most of the sinusoids are contaminated with noise in real life applications. Discrete Fourier Transform is one of the most popular methods to estimate the frequency of a tone contaminated with noise.

Let the signal

$$X = A \cos(w_0 * t) + \epsilon(t)$$

be a continuous single frequency tone. Here, A is the signal amplitude, w_0 is the frequency of the signal, and $\varepsilon(n)$ is the additive white Gaussian noise with zero mean and variance σ^2 .

For estimation of the frequency it is assumed that all the parameters above are unknown. For optimum result, the estimator should be accurate enough, unbiased and computationally simple to avoid complexity.

The idea here is to estimate the frequency of a tone using three DFT samples X_k , X_{k-1} , and X_{k+1} . By using the DFT spectral peak location integer indexes k , $k-1$, $k+1$, correction term $\varepsilon(n)$ can be found and by adding to k spectral location $k_{peak} = k + \varepsilon$ is estimated.

Various methods have been proposed for this single-frequency estimation task. These methods are summarized in the following subsections:

Quadratic Interpolation

This method estimates the correction term by interpolating the three consecutive DFT samples and taking the real part:

$$\varepsilon = -Re \left[\frac{(X_{k+1} - X_{k-1})}{(2X_k - X_{k-1} - X_{k+1})} \right] \quad (1)$$

Quinn's Estimator

This estimator takes three DFT samples around the spectral peak and the correction term is calculated as:

$$\begin{aligned} \alpha_1 &= Re(X_{k-1} / X_k) \\ \alpha_2 &= Re(X_{k+1} / X_k) \\ \delta_1 &= \alpha_1 / (1 - \alpha_1) \\ \delta_2 &= \alpha_2 / (1 - \alpha_2) \\ \text{If } \delta_1 > 0 \ \& \ \delta_2 > 0 \ \varepsilon &= \delta_2 \ \text{Else } \varepsilon = \delta_1 \end{aligned} \quad (2)$$

Macleod's Estimator

This estimator takes the isolated spectral peak sample and calculates the correction term as:

$$R = Re[(X_{k-1} \cdot X_k^*) (X_k \cdot X_k^*) (X_{k+1} \cdot X_k^*)]$$

$$\gamma = \frac{R(1) - R(3)}{2R(2) + R(1) + R(3)}$$

$$\varepsilon = \left(\left(\sqrt{1 + 8\gamma^2} \right) - 1 \right) / 4\gamma \quad (3)$$

III. RESULTS AND DISCUSSION

In order to evaluate performances of the estimators, a sinusoid single frequency tone signal is employed in the experiments. A zero mean white Gaussian noise with various signal-to-noise (SNR) ranging from 0 dB to 10 dB is added to noise-free signal. 64-point FFT is applied to all resulting noisy signals. Tests are iterated for 1000 times.

Robustness to Noise

For a pilot spectral peak at bin location $n = 9.5$, the performances of the estimators against varying SNR from 0 dB to 10 dB are depicted in Figure 1 and Figure 2.

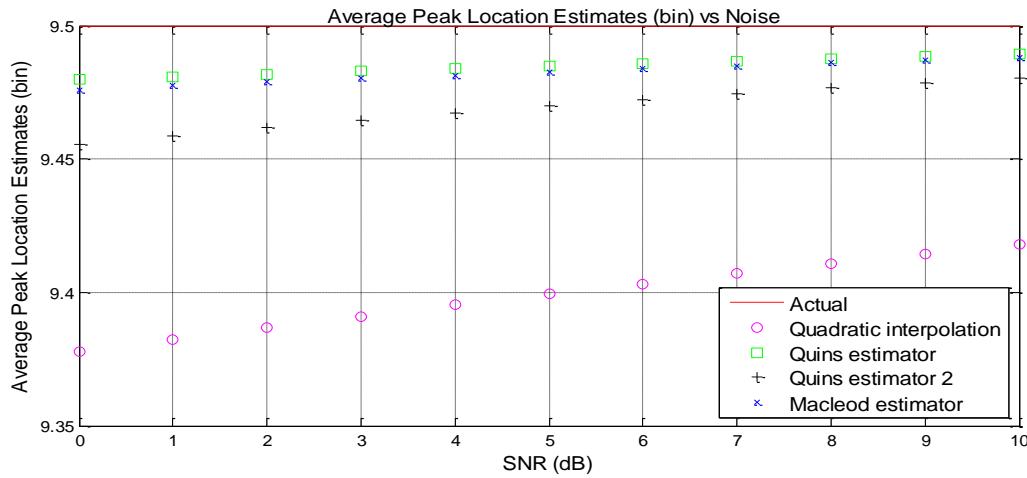


Figure 1. Average peak location estimates against varying SNR from 0 to 10 dB

Figure 1 shows the average peak location estimates of the corresponding methods in this case. As shown in Figure 1, the best performance is given by the Quin’s first estimator (will be called Quin1 hereafter) and Macleod estimator is very close to the Quin1 whilst the actual bin is 9.5. Accuracy of both estimators is around 9.48.

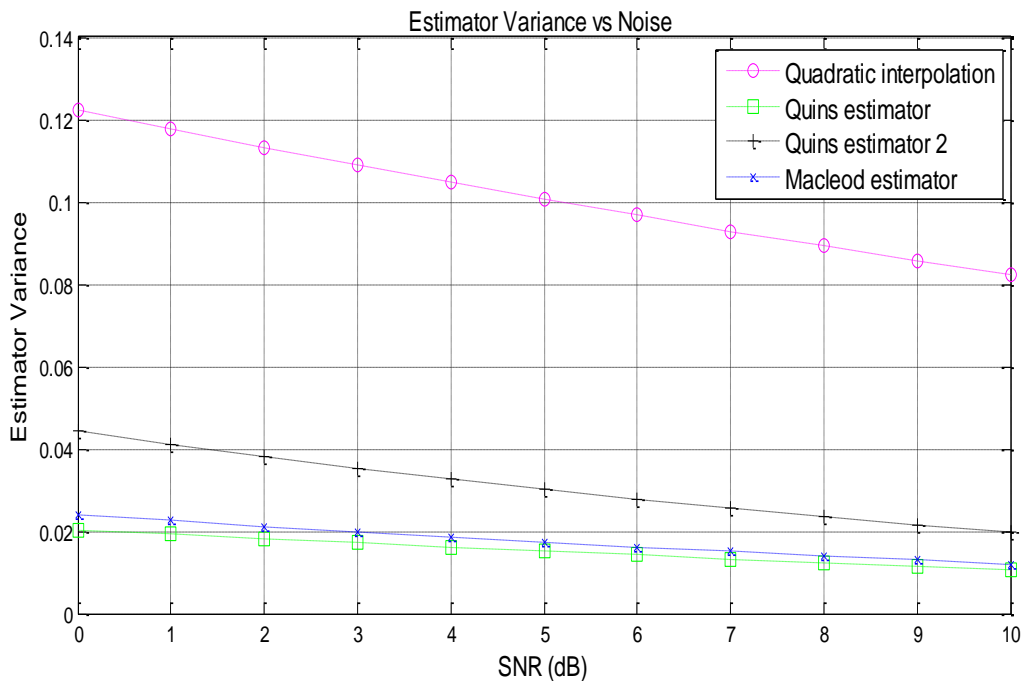


Figure 2. Estimator variances against increasing SNR from 0 to 10 dB

In terms of estimator variances, as seen in Figure 2, the accuracy of Quin1 estimator is very high against varying SNR. The variance of the estimator is less than 0.02. At high SNR values, the performance of Macleod estimator is comparable to Quin1 estimator. These two estimators outperform the rest of the estimators in this case. The results of estimator variances for Quin1 and Macleod estimators verify the results of average peak location estimations.

Robustness to Imperfections in signal frequency

Figure 3 and Figure 4 show the results of estimator performances when the spectral peak bin location is varied from 9 to 10. In this case, SNR is kept constant at 3 dB.

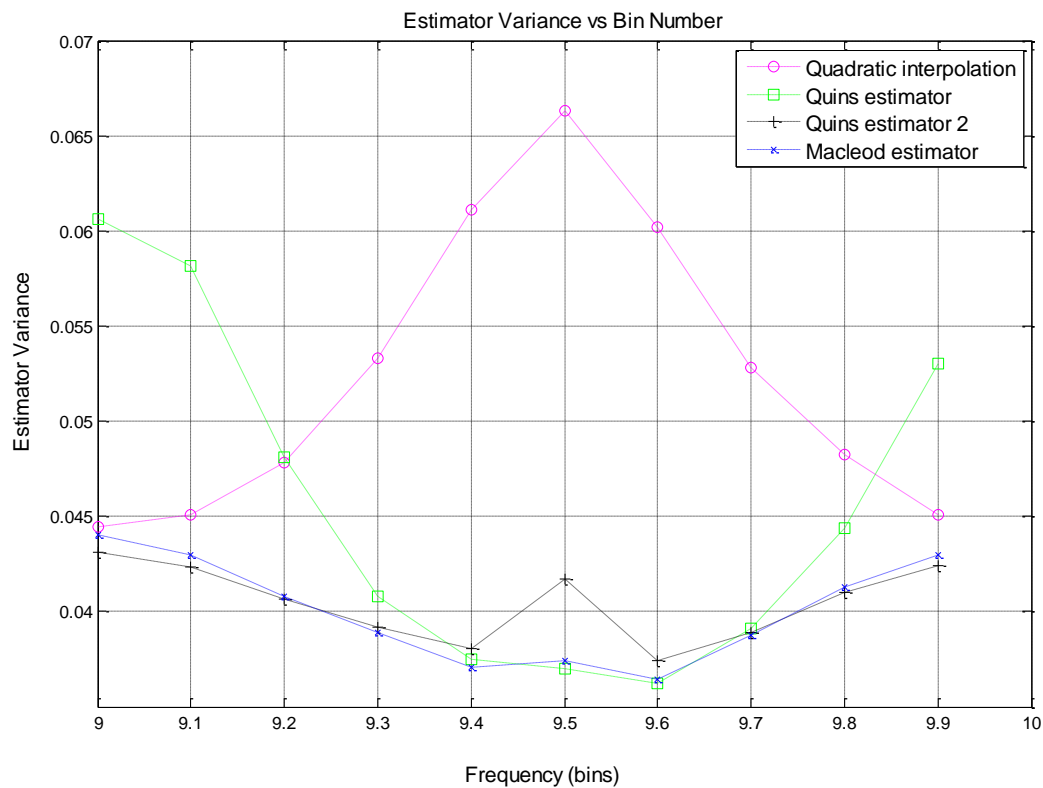


Figure 3. Estimator variances against varying spectral bins from 9 to 10

Figure 3 shows estimator variances of the corresponding methods in this case. Starting from bin 9, in contrast to SNR case, it can be observed that Quin1 estimator generally performs poorer than other estimators except quadratic interpolator. In this case, Macleod estimator achieves the lowest and most stable variance values. At bin 9.5, which is the half-way of the bin range, all of the estimators except quadratic interpolator give satisfying results. Their variances are less than 0.05. Estimator variance is at maximum for quadratic interpolator at bin 9.5.

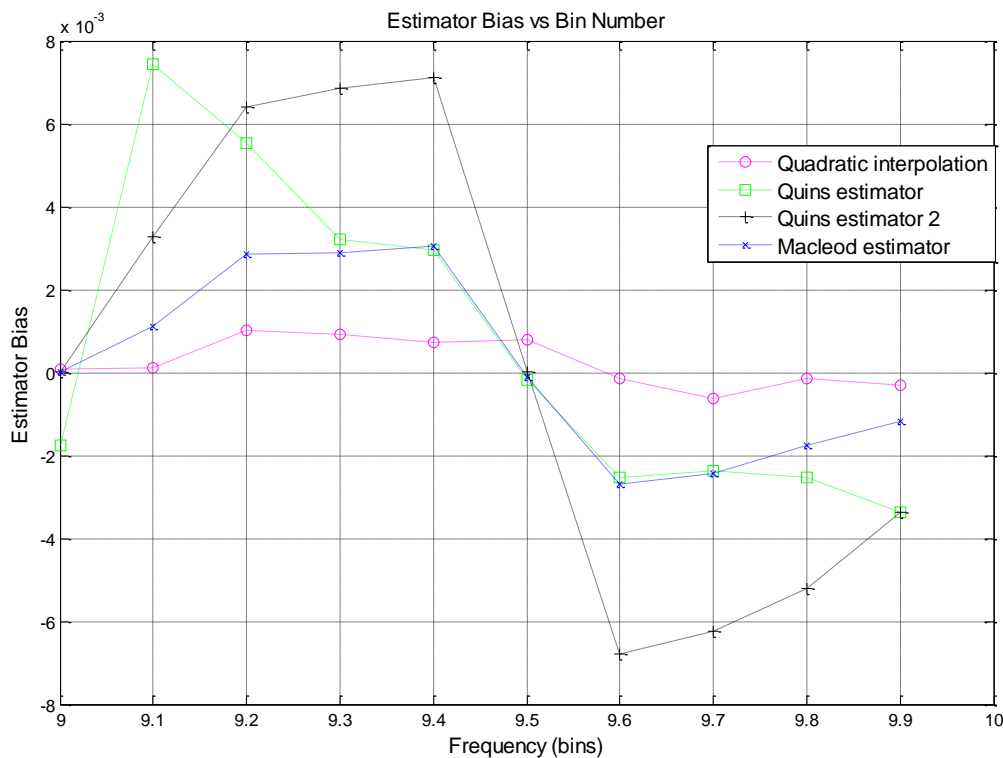


Figure 4. Estimator biases against varying spectral bins from 9 to 10.

In terms of estimator bias, it is clear from Figure 4 that quadratic interpolator achieves most stable results and outperforms the other estimators when the peak location deviates from bin 9.5 towards to 0 and 10. Quin's estimators overshoot and undershoot towards to bin 9.5 and their performances are relatively worse than the other estimators.

IV. CONCLUSION

In this study, we have presented performance results of various estimators for the frequency estimation task of single-tone signals. Estimators have been tested against varying signal-to-noise (SNR) ratio and imperfections in signal frequency. Experimental results have shown that Quin's estimator is more robust to noise and outperforms the Macleod and quadratic interpolation estimators in case of varying SNR. On the other hand, in the case of imperfections in signal frequency, although Macleod estimator have obtained lowest and stable results in terms of estimator variance, in terms of bias quadratic interpolator has achieved the most stable results.

V. REFERENCES

- [1] D. B. Percival and A. T. Walden, *Spectral Analysis for Physical Applications*, Cambridge Univ. Press, 1993.
- [2] S. M. Kay, *Modern Spectral Estimation*. Englewood Cliffs, 1988.
- [3] L. C. Palmer, "Coarse frequency estimation using the discrete fourier transform", *IEEE Trans. Inf. Theory*, vol. IT-20, pp. 104–109, 1974.
- [4] D. Rife and R. Boorstyn, "Single-tone parameter estimation from discrete-time observations", *IEEE Trans. Inform. Theory*, vol. IT-20, no. 5, pp. 591–598, 1974.
- [5] B.G. Quinn, "Estimating frequency by interpolation using Fourier coefficients", *IEEE Trans. Signal Processing*, vol. 42, no. 5, pp. 1264–1268, 1994.
- [6] M. Macleod, "Fast nearly ML estimation of the parameters of real or complex single tones or resolved multiple tones", *IEEE Trans. Signal Processing*, vol. 46, no. 1, pp. 141–148, 1998.
- [7] T. Grandke, "Interpolation algorithms for discrete Fourier transforms of weighted signals", *IEEE Trans. Instrum. Meas.*, vol. IM-32, pp. 350–355, 1983.



-
- [8] P. Voglewede, "Parabola approximation for peak determination", *Global DSP Mag.*, vol. 3, no. 5, pp. 13-17, 2004.
- [9] E. Jacobsen, "On local interpolation of DFT outputs", <http://www.ericjacobsen.org/FTinterp.pdf>.

CITE AN ARTICLE

Baykal , M., & Gençol, K. (n.d.). PERFORMANCE EVALUATION OF SINGLE-TONE FREQUENCY ESTIMATORS UNDER NOISY AND IMPERFECT SIGNAL CONDITIONS . *INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY*, 6(12), 37-42.